Enrollment No. ____________

**Shree Manibhai Virani and Smt. Navalben Virani Science College (Autonomous), Rajkot**
Affiliated to Saurashtra University, Rajkot

**SEMESTER END EXAMINATION APRIL - 2018**

## M.Sc. Mathematics

**16PMTDC07 – MATHEMATICAL STATISTICS**

***Duration of Exam – 3 hrs*** ***Semester – IV*** ***Max. Marks – 70***

***Part A* (5X2= 10 marks)**
Answer **ALL** questions

1. Find arithmetic mean and geometric mean of first five natural numbers.
2. Define Mutually exclusive events. If two possible events A and B are mutually exclusive then find $P(A \cap B)$
3. Let X be a random variable with probability density function f(x) and cumulative distribution function F(x). True or False?
   a. f(x) can't be larger than 1.
   b. F(x) can't be larger than 1.
4. What is the probability that a person flipping a fair coin requires four tosses to get a head?
5. Define Null and Alternative Hypothesis

***Part B* (5X5 = 25 marks)**
Answer **ALL** questions

6a. State and prove Baye's theorem for conditional probability.

**OR**

6b. Sixty percent of new drivers have had driver education. During their first year, new drivers without driver education have probability 0.08 of having an accident, but new drivers with driver education have only a 0.05 probability of an accident. What is the probability a new driver has had driver education, given that the driver has had no accident the first year?

7a. If X is uniform on the interval $[a, b]$ then the mean, variance and moment generating function of X are given by

$$E(x) = \frac{a+b}{2}$$

$$Var(x) = \frac{(b-a)^2}{12}$$

$$M(t) = \frac{e^{ta} - e^{tb}}{t(b-a)}, \text{if } t \neq 0$$

**OR**

7b. If X is binomial random variable with parameters p and n, then the mean, variance and moment generating functions are respectively given by

$$\mu_x = np$$

$$\sigma^2{}_x = np(1-p)$$

$$M_x(t) = [(1-p) + pe^t]^n$$

8a. Let $X_1, X_2, \dots, X_n$ be a random sample from a normal population with mean μ and variance $\sigma^2 > 0$. If $\overline{X} = \frac{\sum X_i}{n}$ and $V^2 = \frac{1}{n}\Sigma (X_i - \overline{X})^2$ and $X_{n+1}$ is an additional observation, what is the value of m so that the statistics $\frac{m(\overline{X} - X_{n+1})}{V}$ has t-distribution.

**OR**

8b. Define maximum likehood estimator. If $X_1, X_2, \dots, X_n$ is a random sample from a distribution with density function

$$f(x,\theta) = \begin{cases} \frac{1}{\theta}, & if\ 0 < x < \theta \\ 0, & otherwise \end{cases}$$

then what is the maximum likelihood estimator of

9a. A company is interested in finding out if there is any difference in average salary received by managers of two divisions. Accordingly samples of 12 managers in the first division and 10 managers in the second were selected at random. The result are given below :

| | I division | II division |
|---|---|---|
| Sample size | 12 | 10 |
| Average Monthly Salary | 12500 | 11200 |
| Sample Standard Deviation | 320 | 480 |

Is there any significant difference between average salary received by managers of two divisions,

**OR**

9b. 12 students were given intensive coaching and 5 tests were conducted in a month. The score of 1st and 5th tests are given below. Does the scores in 1 to $5^{th}$ test shows improvement?(Take $\alpha = 0.01$)

| No Of Students | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Mark In 1st Test | 50 | 42 | 51 | 26 | 35 | 42 | 60 | 41 | 70 | 55 | 62 | 38 |
| Mark in 5th Test | 62 | 40 | 61 | 35 | 30 | 52 | 68 | 51 | 84 | 63 | 72 | 50 |

10a. A die is thrown 150 times and the following results are obtained.

| Number turned up | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Frequency | 19 | 23 | 28 | 17 | 32 | 31 |

Test the hypothesis that the die is unbiased at 5 % level of significance

**OR**

10b. A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured a third class, 90 were placed in second class and 20 got a first class. Are these figures commensurate with the general examination result which is the ratio of 4 : 3 : 2 : 1 for the various categories respectively?

## ***Part C*** ***(5X7 = 35 marks)***

Answer **ALL** questions

11a. If $X \sim GAM(\theta, \alpha)$, then

$$E(X) = \theta\alpha$$

$$\sigma^2_X = \theta^2\alpha$$

$$M_X(t) = \left(\frac{1}{1-\theta t}\right)^\alpha \ if\ t < \frac{1}{\theta}$$

**OR**

11b. If $X \sim N(\mu, \sigma^2)$ then

$$E(X) = \mu$$
$$\sigma^2_X = \sigma^2$$
$$M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

12a. Suppose $x_1, x_2, \cdots, x_n$ is a random sample from a population X with density

$$f(x, \alpha, \beta) = \begin{cases} \dfrac{1}{\alpha - \beta}, & \text{if } \alpha < x < \beta \\ 0, & \text{Otherwise} \end{cases}$$

Find the estimators of $\alpha$ and $\beta$ by the moment method.

**OR**

12b. Let $X_1, X_2, \cdots, X_n$ be a random sample of size n from normal population known mean $\mu$ and variance $\sigma^2 > 0$. Show that $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is an unbiased estimator of $\sigma^2$. Further, show that $S^2$ cannot attain the Cramer-Rao lower bound.

13a. State and prove Chebyshev's inequality.

**OR**

13b. State and prove Cramer Rao lower bound for variance of estimators.

14a. i) In a sample of 500 people from a village in Rajasthan 280 were found to be rice eaters and rest of them are wheat eaters. Can we assume that both food articles are equally popular(Take α = 0.05)
ii) A sample of 400 male students, it is found to have a mean height of 171.38 cm. Can it be reasonably regarded as a sample from a large population with mean height 171.17 cm and population standard deviation 3.30 cm?(Take α = 0.05)

**OR**

14b. i) A random sample of 100 mill workers at Kanpur showed their mean wage to be Rs 350 with a population standard deviation of Rs 28. Another random sample of 150 mill workers in Nagpur showed wage to be Rs 390 with a population standard deviation of Rs 40.Do the mean wages of workers in Nagpur and Kanpur differ significantly? Use 0.05 level of significance
ii) Ten oil tins are taken at random from an automatic filling machine. The mean weight of 10 tins is 15.8kg with a standard deviation of 0.5 kg. Does the sample mean differ significantly from the intend weight of 16 kg?
(Take α = 0.05)

15a. Explain Type-I and Type-II errors with examples.

**OR**

15b. Explain rejection and non rejection areas with examples.